A Review of the unit fraction method of reciprocal adjustment recently uncovered in Egyptian metrology.

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ABSTRACT:

Essential ideas on the method of reciprocal adjustment are considered in detail by deriving all the relevant parametric variables that are needed in the construction and the re-survey of a monument based on this particular type of unit fraction metrological practice when used to encode metrical parameters in the structure.

1 Introduction

The thesis: The central thesis is that the ancient Egyptian builders of The Great Pyramid used a particular method involving their usual *unit fractions* in their metrology, that enabled them to encode the metrical attributes of a length, L, that is, the unit, u, and the count, n, for which, $L = n \cdot u$, by constructing two *similar* lengths, L and L', which differ from each other by only a small amount, using the reciprocal of the count, $1/n$, so that a future survey could later readily decode these metrical values. The evidence for this was presented in a previous paper,[(1)] where these formulas were first introduced:

$$L = n \cdot u \tag{1}$$
$$L' = (n - 1/n) \cdot u \tag{2}$$

$$\Delta L(n) = L - L' = u/n \tag{3}$$

$$n = 1/\sqrt{(1 - L'/L)} \tag{4}$$
$$u = \sqrt{L \cdot (L - L')} \tag{5}$$

As it turns out, on further examination, the builder employing this system would only need to consider three metrics, based on the reciprocals of the *linear*, the *square*, and the *cube*, of the count, n, since the *unit*, the *delta*, and the *tolerance*, are precisely given, or simply estimated, by the rules,

$$u = \frac{L}{n} \quad , \quad \Delta L = \frac{L}{n^2} \quad , \quad tol = \frac{L}{n^3} \tag{6}$$

as this triplet is all that is necessary and sufficient to plan the construction.

10^0	10^1	10^2	10^3	10^4	10^5	10^6
1	10	100	1,000	10,000	100,000	1,000,000

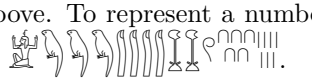
The ancient Egyptians used a decimal system to write numerical values, and the Egyptian Hieroglyphs for these numerical values are constructed from 7 numbers that are powers of 10 as shown here above. To represent a number like 1,342,157, for example, the scribe would write  .

Figure 1: *The two-fingers hieroglyph, representing both the numerical value 20,000 and concepts like accurate and precise.*

To record the division of a length, L, into 28 parts, for example, requires,

$$u = \frac{L}{28} \quad , \quad \Delta L = \frac{L}{28^2} = \frac{L}{784} \quad , \quad tol = \frac{L}{28^3} = \frac{L}{21,952} \quad , \tag{7}$$

so that the *tolerance* required for the measuring tools would be on the order of that very same 1 in 20,000 parts of the Length, which just so happens to be the special hieroglph of two-fingers, , used in the ancient Egyptian sign for the number 20,000 and also used as a determinative in Egyptian words for those very concepts indicating accuracy and precision.

Figure 2: *Two-Finger use in 'Accurate' (left) and 'Precise' (right).*
Egyptian Hieroglyhs for the words `accurate' and `precise,'
showing the use of the `two-finger' glyph as determinative
in the word construction. See Gardner ``Egyptian Grammar''
sign-list pg.456 [Ref-(2)]. And for `exact' and related terms,
see Budge ``Hieroglyphic Dictionary'', 139, 332-3, [Ref-(3)].

As we have seen in our previous work,(1) the builders of the Great Pyramid repeatedly used this particular count, $n = 28$, in various parts, like the King's Chamber, the Socket Square, the Grand Gallery count of mortices on the side ramps, and, of course, in the standard measuring Rod which Egyptologists now call the Royal Cubit which is divided into 7 palms of 4 digits each, or 28 digits. So, the use of the two-finger hieroglyph, $\equiv 20{,}000$, in reference to ideas of accuracy and precision is in agreement with the *tolerance* required for their apparently favored design count, $tol = L/28^3 = L/21,952 \approx L/20,000$.

What tolerance?:

To measure the dimensions of the walls in the King's Chamber of the Great Pyramid at Giza, the pre-construction engineers and subsequent post-construction surveyors, both need to consider the accuracy of their measuring rulers as required to endode and decode the metrics.

Since, the North Wall has length, L = 412.14″, at the ceiling level, and is divided into 40 parts, the ruler must have sufficient accuracy to establish the tolerance of, $tol = L/40^3 = 412.14''/64,000 = .00643'' = 1''/155$, and since the East Wall has width, W = 206.30″, at the ceiling level, and is divided into 28 parts, the ruler must have accuracy to establish the corresponding tolerance of, $tol = W/28^3 = 206.30''/21,952 = .00940'' = 1''/106$.

In 1883, when Petrie published his report(4) on this room's survey, he reportedly used measuring rulers graduated into 100ths of an inch, $.01'' = 1''/100$, whose graduation marks were known to 1000ths of an inch, $.001'' = 1''/1000$, so his measuring instruments were right at the borderline of that required to determine the count, $n = 28$, of the East and West wall dimensions, $1''/100 \sim 1''/106$; while being somewhat more inadequate to measure the North and South wall dimensions, $1''/100 > 1''/155$, to determine the count, $m = 40$, with the required confidence; and as a result of this, even though Petrie's results are the best we have of the dimensions of this Chamber, to date, the measurements are not known with sufficient accuracy to conclude that the original intent of the architect of this pyramid was indeed to divide that room by the indicated counts, 28 and 40, beyond all possible doubt.

2 Analytics

The builder's target length, L, we refer to as the *reference* length, while the shortened alternative length, L', used to compare, we name the *apposite.* Typically, the *opposite* wall or the *opposite parallel* side of a rectangle is the *apposite*, but not necessarily so, as we have seen in the Great Pyramid, where the *adjacent* side or the *crossing diagonal* of the socket square could also play the role of the alternative comparing length. The difference between these two is the target *delta*, ΔL.

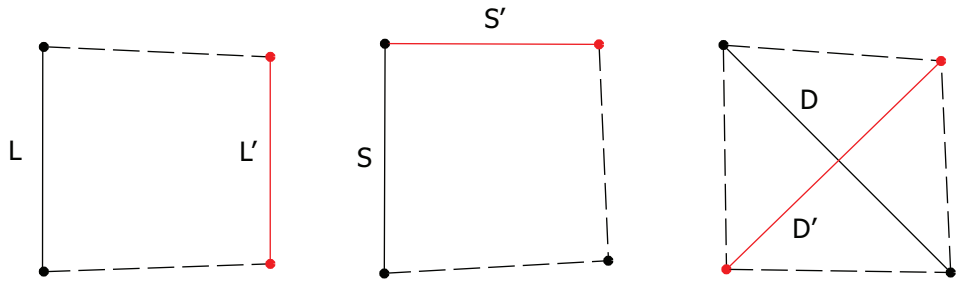


Figure 3: *Three types of "apposites" on a nominal square.*

On the left – "parallel apposites," $L'/L = (1 - 1/n^2)$*; e.g. GP:* $n = 28$*.*
In the middle – "adjacent apposites," $S'/S = (1 - 1/n^2)$*; e.g. GP:* $n = 90$*.*
On the right – "cross-diagonal apposites," $D'/D = (1 - 1/n^2)$*; e.g. GP:* $n = 25$*.*

To ensure the construction is precise enough to enable any future survey to distinguish between the encoded count, n, and its nearby values, $n+1$ or $n-1$, the builder needs to carefully consider the *upper range*, δ_+, and *lower range*, δ_-, that measures the distance between these alternatives that typically partition the *full range*, rn, into *unequal halves*—using somewhat loose language—since they are only *almost halves*, but actually differ slightly one side from the other.

Once the builder keeps his constructed delta, ΔL, within $1/2$ of the *upper range*, which becomes his effective *tolerance*, tol_+, and within $1/2$ of the *lower range*, which is his *tolerance*, tol_-, he can be sure the future surveyor has a good chance to decode the *unit*, u, and the *count*, n, in the reference length, $L = n \cdot u$.

reference	L	$= n \cdot u$
apposite	L'	$= (n - 1/n) \cdot u$
delta	ΔL	$= L - L' = u/n = L/n^2$
upper range	δ_+	$= L/(n-1)^2 - L/n^2$
lower range	δ_-	$= L/n^2 - L/(n+1)^2$
full range	rn	$= L/(n-1)^2 - L/(n+1)^2$
upper tolerance	tol_+	$= 1/2 \cdot \delta_+ = 1/2 \cdot (L/(n-1)^2 - L/n^2)$
lower tolerance	tol_-	$= 1/2 \cdot \delta_- = 1/2 \cdot (L/n^2 - L/(n+1)^2)$
min tolerance	$min(\frac{1}{2}\delta_-, \frac{1}{2}\delta_+)$	$= 1/2 \cdot \delta_- = 1/2 \cdot (L/n^2 - L/(n+1)^2)$
flatness	tol_0	$= 1/2 \cdot (1/2 \cdot \delta_-) = 1/4 \cdot (L/n^2 - L/(n+1)^2)$

However, when a survey is done of an existing construction, the surveyor's measuring error is typically *symmetrical* about his mean values, so he would have, $\Delta L = meas\,\langle \Delta L \rangle \pm err$, and here the builder could assist the effort by making sure his construction is within the smaller of the two tolerance ranges, $min(1/2 \cdot \delta_-, 1/2 \cdot \delta_+)$, which is easily proven to be the left side, tol_-; thus he could aim the construction to fit, $\Delta L \pm 1/2 \cdot \delta_-$. Finally, when parallel walls of a room are being used to define the limits, the roughness of the surface of the alternate walls also contribute to the error of placement and re-measurement, hence they must be dressed and polished to a flatness, tol_0; see the formulas in the table above.

To show that, $min(\frac{1}{2}\delta_-, \frac{1}{2}\delta_+) = \frac{1}{2}\delta_-$, it suffices to prove, $\delta_+ > \delta_-, \forall\, n > 1$:

$$\begin{aligned}
\delta_+/\delta_- &= \frac{L/(n-1)^2 - L/n^2}{L/n^2 - L/(n+1)^2} && (8)\\
&= \frac{(n+1)^2(2n-1)}{(n-1)^2(2n+1)} && (9)\\
&= \frac{2n^3 + 3n^2 - 1}{2n^3 - 3n^2 + 1} && (10)\\
&= \frac{2n^3 - 3n^2 + 1 + 6n^2 - 2}{2n^3 - 3n^2 + 1} && (11)\\
&= \left(1 + \frac{2(3n^2-1)}{(n-1)^2(2n+1)}\right) \quad > \quad 1, \quad \forall\, n > 1, \ \text{Q.E.D} && (12)
\end{aligned}$$

Regardless of what the builder does to facilitate measurements by the future surveyor in his effort to duplicate the metrics, the surveyor's measurement must fall within the range;

$$\frac{L}{n^2} - \frac{1}{2}\delta_- < meas\,\langle \Delta L(n) \rangle < \frac{L}{n^2} + \frac{1}{2}\delta_+ \qquad (13)$$

and by substitution of the tolerances, $1/2 \cdot \delta_-$ and $1/2 \cdot \delta_+$, this amounts to,

$$\frac{1}{2}\left(\frac{L}{n^2} + \frac{L}{(n+1)^2}\right) < meas\,\langle \Delta L(n) \rangle < \frac{1}{2}\left(\frac{L}{(n-1)^2} + \frac{L}{n^2}\right) \quad (14)$$

We can locate the midpoint of this range, $\Delta L(n; \text{MID})$, and the symmetric tolerance about this midpoint, $tol(n; \text{MID})$, to find the optimum equivalent symmetrical parameters;

$$\Delta L(n; \text{MID}) = \frac{1}{2}\left[\frac{1}{2}\left(\frac{L}{n^2} + \frac{L}{(n+1)^2}\right) + \frac{1}{2}\left(\frac{L}{(n-1)^2} + \frac{L}{n^2}\right)\right] \quad (15)$$

$$tol(n; \text{MID}) = \frac{1}{2}\left[\frac{1}{2}\left(\frac{L}{(n-1)^2} + \frac{L}{n^2}\right) - \frac{1}{2}\left(\frac{L}{n^2} + \frac{L}{(n+1)^2}\right)\right] \quad (16)$$

$$\langle \Delta L(n) \rangle = \Delta L(n; \text{MID}) \pm tol(n; \text{MID}) \quad (17)$$

This gives us the 'widest' possible symmetrical tolerance, $tol = tol(n; \text{MID})$, that can be used in construction, to help ensure the future surveyor's measurement will land somewhere within the required range and find the correct values for count, n, and unit, u, that compose the length, $L = n \cdot u$.

$$\Delta L(n; \text{MID}) - tol < meas\,\langle \Delta L(n) \rangle < \Delta L(n; \text{MID}) + tol \quad (18)$$

The builder shifts the target value for the $\Delta L(n)$, slightly to the right, aiming for that midpoint, $\Delta L(n; \text{MID})$, so that the tolerance can be widened within the same end points of the fixed tolerance range. Now, the surveyor finds;

$$meas\,\langle \Delta L(n) \rangle = \Delta L(n; \text{MID}) \cdot \left(1 \pm \frac{tol(n; \text{MID})}{\Delta L(n; \text{MID})}\right) \quad (19)$$

$$= \Delta L(n; \text{MID}) \cdot \left(1 \pm \frac{1}{N}\right) \quad (20)$$

$$
\begin{aligned}
N &= \frac{(1/n^2 + 1/(n+1)^2) + (1/(n-1)^2 + 1/n^2)}{(1/(n-1)^2 + 1/n^2) - (1/n^2 + 1/(n+1)^2)} && (21)\\
&= \frac{2/n^2 + 1/(n+1)^2 + 1/(n-1)^2}{1/(n-1)^2 - 1/(n+1)^2} && (22)\\
&= \frac{2(n-1)^2(n+1)^2 + n^2(n-1)^2 + n^2(n+1)^2}{n^2(n+1)^2 - n^2(n-1)^2} && (23)\\
&= \frac{4n^4 - 2n^2 + 2}{4n^3} && (24)\\
&= n - \frac{1}{2n} + \frac{1}{2n^3} && (25)\\
&\approx n && (26)
\end{aligned}
$$

So, for sufficiently large count, n, we may readily approximate, $N \approx n$, with no loss of generality, since the tolerance is a ballpark figure, which is not required to be an exact metric. Similarly, we may substitute, n for $n+1$, and substitute, n for $n-1$, for sufficiently large, n, to further simplify the tolerance figure as follows;

$$
\begin{aligned}
tol(n;\text{MID}) &= \frac{1}{N} \cdot \Delta L(n;\text{MID}) && (27)\\
&\approx \frac{1}{n}\frac{1}{2}\left[\frac{1}{2}\left(\frac{L}{n^2} + \frac{L}{(n+1)^2}\right) + \frac{1}{2}\left(\frac{L}{(n-1)^2} + \frac{L}{n^2}\right)\right]\\
&\approx \frac{1}{n}\frac{1}{2}\left[\frac{1}{2}\left(\frac{L}{n^2} + \frac{L}{n^2}\right) + \frac{1}{2}\left(\frac{L}{n^2} + \frac{L}{n^2}\right)\right]\\
&\approx \frac{L}{n^3} && (28)
\end{aligned}
$$

This tells us that the widest symmetrical construction tolerance achievable is on the order of, $tol = L/n^3$.

We can compare this estimate of tol with the more exact value, $tol(n;\text{MID})$, as given in equation (16) above, in practical examples, such as the counts found in the Great Pyramid, $n = 25, 28, 40, 90$. Since the typical side length of the socket square of the Great Pyramid has length $9,130.8''$ (e.g. East Side), we can consider a practical length, $L = 10,000''$, to examine the difference that would occur in actual practice. The following table shows the results;

n	$tol(n;\text{MID})$	$L = 10,000''$	$tol = L/n^3$	$L = 10,000''$
25	$L/15,575.$	$= \pm 0.642''$	$L/15,625$	$= \pm 0.640''$
28	$L/21,896.$	$= \pm 0.457''$	$L/21,952$	$= \pm 0.456''$
40	$L/63,920.$	$= \pm 0.156''$	$L/64,000$	$= \pm 0.156''$
90	$L/728,820.$	$= \pm 0.0137''$	$L/729,000$	$= \pm 0.0137''$

So, our measuring rod must have a resolution able to detect a difference of about $\frac{1}{2}''$, if we divide a length of, $L = 10,000''$, by 28; and must detect about, $\frac{1}{6}''$, to divide by 40; and have precision able to detect about, $\frac{1}{72}''$, if that length is divided by 90. If we can measure to, $\frac{1}{100}''$, that is, $\pm.01''$, all these practical counts used at the base of the Great Pyramid are covered. Unfortunately, Petrie measures the socket square to a resolution of $\frac{1}{10}'' = .1''$, with an 'estimated' resultant error from computation of $\pm.3''$, so that his measures are right up at the borderline precision required to measure the Great Pyramid. His measures just make it within range for some values, like the East and West sides, which are divided by the count, $n = 28$, since, $|\pm .3''| < |\pm .456''|$; and fall just outside the range for others, like the North and South sides, which are divided by the count, $n = 40$, where, $|\pm .3''| > |\pm .156''|$. Were it not for the clever thoughtful *check* placed in the subterranean chamber, by the builder, that allows us to recognise and correct this error, we wouldn't have been able to detect and fix the problem by analysing the structure. Now lets see how far the builder must shift the $\Delta L(n)$ to hit that midpoint target, $\Delta L(n;\text{MID})$.

$$\Delta L(n;\text{MID}) = \frac{L}{n^2} + \frac{1}{4}\left[\frac{L}{(n+1)^2} + \frac{L}{(n-1)^2} - 2\frac{L}{n^2}\right] \tag{29}$$

$$= \Delta L(n) + \frac{1}{4}\left(\delta_+ - \delta_-\right) \tag{30}$$

Again, consider the counts, $n = 25, 28, 40, 90$, used in the GP, with a typical length, $L = 10,000''$, along with the symmetrical *tol*, to get an idea of the measures;

n	$\Delta L(n;\text{MID})$	$= \Delta L(n)$	$+ 1/4 \cdot (\delta_+ - \delta_-)$	$tol = L/n^3$
25	$16.038\,503''$	$= 16.000\,000''$	$+ 0.038\,503''$	$\pm 0.640''$
28	$12.779\,558''$	$= 12.755\,102''$	$+ 0.024\,456''$	$\pm 0.456''$
40	$6.255\,865''$	$= 6.250\,000''$	$+ 0.005\,865''$	$\pm 0.156''$
90	$1.234\,797''$	$= 1.234\,568''$	$+ 0.000\,229''$	$\pm 0.0137''$

Clearly, the tolerance metric, *tol*, is significantly larger than the adjustment of the delta to mid value, $|tol| >> (\Delta L(n;\text{MID}) - \Delta L(n))$, suggesting that while the builder might be in posession of such accurate measuring instruments to target the MID, the future surveyor can reasonably ignore this small adjustment in his work, and simply take the position he's trying to find the $\Delta L(n) = L/n^2$,

being more concerned his own error, $\Delta L(n) = meas\,\langle \Delta L(n) \rangle \pm err$, is within the symmetrical tolerance, $|\pm err| < |\pm tol|$, of the theory.

Considering the Royal Cubit Rod was divided into, $7 \times 4 \times 16 = 28 \times 16 = 7 \times 64 = 448$, parts, with Petrie's measurement of $20.632''$ for this rod, the *resolution* of the Royal Cubit is typically, $20.632''/448 = 0.046''$, which is sufficient for the three GP counts, $n = 25, 28, 40$. But, $n = 90$, is beyond the Cubit Rod resolution. That would require the division of a digit, d, two levels lower on the usual half-scale, $d/16 = 0.046''$, $d/32 = 0.023''$, $d/64 = 0.012''$, and now we're down to the Horus Eye fraction limit, $1/64$, of a digit.

If we define the adjustment to MID, as the, $shift = (\Delta L(n; \text{MID}) - \Delta L(n))$, then the size of this *shift* can be compared to the symmetrical tolerance, *tol*, using the more exact formulas, for a given count, n,

$$\frac{tol}{shift} = \left[\frac{\frac{1}{(n-1)^2} - \frac{1}{(n+1)^2}}{\frac{1}{(n+1)^2} + \frac{1}{(n-1)^2} - \frac{2}{n^2}} \right] \tag{31}$$

$$= 16.7 \quad , \quad (\text{when}, n = 25) \tag{32}$$

$$= 18.7 \quad , \quad (\text{when}, n = 28) \tag{33}$$

$$= 26.7 \quad , \quad (\text{when}, n = 40) \tag{34}$$

$$= 60.0 \quad , \quad (\text{when}, n = 90) \tag{35}$$

Typically, then, the *shift* required by the builder, to assist the future survey, is more than 10 times smaller than the symmetrical tolerance, $tol > 10 \times shift$, so that, relative to that ballpark figure for tollerable error of measurement, the accounting for the *shift* can be neglected by that future surveyor.

This leaves us with that reasonable triple parameter set, of the *unit*, the *delta*, and the *tolerance*, given by,

$$u = \frac{L}{n} \quad , \quad \Delta L = \frac{L}{n^2} \quad , \quad tol = \frac{L}{n^3} \tag{6}$$

The measuring instruments required for the construction and subsequent survey must both be capable of measuring to a precision level given by, $tol = L/n^3$; the measured-back value must thus be in the range,

$$\frac{L}{n^2} - \frac{L}{n^3} < meas\,\langle \Delta L(n) \rangle < \frac{L}{n^2} + \frac{L}{n^3} \tag{36}$$

Of course, there are other considerations, such as how much of the tolerance must be "split" between the construction effort and the subsequent survey.

$$\text{builder error} + \text{surveyor error} \leq tol \tag{37}$$

How much of the *tol* available should we attribute to the builder's error?

The tolerances required in the external socket square are much looser than those of the internal King's Chamber where it even becomes necessary to carefully dress and polish walls to meet the specification.

The Width, W, and Length, L, parameter values of the King's Chamber are about, $W = 200''$ and $L = 400''$, and divided by the counts, $n = 28$ and $n = 40$, respectively, so using these values to get a sense of what's involved we have,

	Width (W)				Length (L)		
u	$200''/28$	$= 7.142\,857''$		u	$400''/40$	$= 10.000\,000''$	
ΔL	$200''/28^2$	$= 0.255\,102''$		ΔL	$400''/40^2$	$= 0.250\,000''$	
tol	$200''/28^3$	$= 0.009\,111''$	$\approx 1''/110$	tol	$400''/40^3$	$= 0.006\,250''$	$\approx 1''/160$
tol_0	$= tol/2$	$= 0.004\,556''$	$\approx 1''/220$	tol_0	$= tol/2$	$= 0.003\,125''$	$\approx 1''/320$

When Petrie measured this room, he used measuring rods with a resolution of one-hundreth of an inch, $1''/100 = 0.01''$, so he could clearly identify values in the neighborhood of the delta, $\Delta L = 0.25''$. However, even though his rod's graduation marks were known to be accurate to one-thousandths of an inch, $1''/1000 = 0.001''$, his instrument's resolution was not sufficient to meet the *tol* specs here. Again, he's on the borderline, with, $1''/100 \approx 1''/110$, barely enabling him to just get the Width count right, $n = 28$; while for the Length count, $n = 40$, his instrument's, $1''/100 > 1''/160$, was somewhat outside the required tolerance that would *guarantee* interpretation of that count. That his data enabled the discovery, $n = 40$, is more by luck, and the fortuitous planning of the builder in using both the ceiling and the floor rectangles to encode the values. All praise to the builder. But, Petrie did do very well with what he had, despite the limitations of his measuring tools.

Looking into the available literature to see what modern investigators found about the builder's capabilities, we find the following statement from the author Chapman,[5]

> *To appreciate reasonably the work of the ancient genius and his willing people, we have to remember that the stones of the pyramid probably averaged over two and a half tons each ; that the polishing of granite was a lost art until about 300 years ago, when it was again brought to life in Aberdeen ; that each of the enormous mass of covering stones, so far as can be judged from presently available evidence, was finished unto the two-hundredth of an inch ;*

pg.7, "The Great Pyramid of Ghizeh. From the Aspect of Symbolism and Religion (1931)," by Frances W. Chapman, ISBN 0-7661-0083-9

Similarly, James Henry Breasted, Professor of Egyptology in the University of Chicago writes,[6] quoting Petrie;

> *The pyramid is or was about four hundred and eighty one feet high, and its square base measured some seven hundred and fifty five feet on a side, but the average error is* ``less than a ten thousandth of the side in equality, in squareness and in level'';[1] *although a rise of ground on the site of the monument prevented direct measurements from corner to corner. Some of the masonry finish is so fine that blocks weighing tons are set together with seams of considerable length, showing a joint of one ten thousandth of an inch, and involving edges and surfaces* ``equal to optician's work of the present day, but on a scale of acres instead of feet or yards of material.''[2]

p.118, "*A History of Egypt - From the Earliest Times to The Persian Conquest*," by James Henry Breasted, Ph.D, Professor of Egyptology and Oriental History in the University of Chicago ; Director of Haskell Oriental Museum ; Director of The Egyptian Expedition of the University of Chicago. New York, Charles Scribner's Sons, 1905 [1] Petrie, History of Egypt, I, p.40 [2] Ibid.

From Flinders Petrie[7] himself,

> *The workmanship greatly varies in different parts. The entrance passage and the casing are perhaps the finest ; the flatness and squareness of the joints being extraordinary,* equal to opticians' work of the present day, but on a scale of acres instead of feet or yards of material. *The squareness and level of the base is brilliantly true, the average error being* less than a ten-thousanth of the side in equality, in squareness, and in level. *The Queen's chamber is also very finely fitted, the joints being scarcely perseptible. Above that the work is rougher ; the grand gallery has not this superlative fineness, and the construction of the King's chamber is flagrantly out of level, though its granite courses are fairly well wrought. A change of design is also shown by the shaft which has been cut through the masonry from the grand gallery to the subterranean parts ; and also by the unfinished rough core masonry left for the floor of the Queen's chamber.*

p.40, Chapter III - The Fourth Dynasty, *A History of Egypt Vol. I. - From the Earliest of Times to the XVIth Dynasty*," by W. M. Flinders Petrie, 1894, Methuen and Co., 36 Essex Street, W.C. LONDON.

So, according to Frances W. Chapman, the flatness of some exposed stone surfaces are at least, $1''/200$. While, according to Prof. James H. Breasted, the thickness of some stone joints are no more than, $1''/10,000$, which would imply the surfaces of the stone blocks that butt together to such a fine "optician's

level" joint—as Prof. W. M. Flinders Petire describes their level of skill—must have been dressed and polished at least to half that figure, so that the Egyptian builders could clearly work stone to a flatness of, $1''/20,000$, at the very least; which again explains the Egyptian two-finger hieroglyph, $\equiv$ 20,000, being used also as a determinative in the words for 'accurate' and 'precise,' for while the Egyptians did not use the British measure of the "inch," as the modern surveyors report on here in their assessment of the monument, nevertheless, their standard "digit" unit was around this size scale in order of magnitude, $1d = .73''$, so that one could equally claim the Egyptians could grind stone to 1 part in 20,000 of a finger width (digit). Hence, these various authors define the practical range that the workmanship exhibits in the Great Pyramid to be anywhere in the following;

$$1''/200 \quad < \quad tol_0 \quad < \quad 1''/20,000 \tag{38}$$

Our theoretical King's Chamber wall flatness tolerance metrics of, $tol_0 \approx 1''/220$, and, $tol_0 \approx 1''/320$, would appear to be completely within their building skill. So, while there are, so far, no direct survey reports on the particular flatness achieved in the King's Chamber itself, they obviously had the skill, as exhibited elsewhere, to prepare the construction to a level of precision way beyond the *tol* requirements, so that the future modern survey might not have to allocate any significant part of the measurement tolerance range to the builders' error, at least from the wall surface variations. Placing blocks requires quite a different skill than polishing walls, so the fact that the builders could polish the blocks to a high as, $tol_0 = 1''/20,000$, says nothing about their ability to place those blocks in just the right spot to get the distance metrics between nominally parallel walls aright.

3 Conclusion

Many archeologists, like Flinders Petrie and Piazzi Smyth, have observed that, in the construction of The Great Pyramid, the builders were not everywhere consistent in the level of precision used and the care taken in perfecting the various parts of the structure. Some parts are roughly constructed, while other parts show a tremendous attention to detail in construction accuracy. From this variability, we can infer that the Egyptians only exhibited and employed their high level of skill and precision when it was absolutely necessary, otherwise they were content to leave things in courser form, saving on time and effort in places. This itself suggests that some parts of the monument were intended to specifically encode metrical parameters that required a high level of tolerance, while other parts did not.

For example, in the Grand Galley channel leading up to the King's Chamber, one can simply "count" the mortices on the ramps to discover that number, 28, appears on either side, there being 27 on each of the parallel ramps, by design

(one is missing on lower West side where the workers cut away the entrance to the escape Well Shaft after construction of the Gallery), and 1 more on the large cubical block at the top of the gallery, that one must step up onto, before the entrance to the King's Chamber. But, once one enters the King's Chamber the only way to discover the numeric count, 28, is there again, in the East and West walls, is to actually "measure" the width of the room, and here the dressing and polishing of the granite walls becomes a critical feature that enables the encoding and decoding of this same numeric reference, 28, placed there again by the builders.

Using the *principle of reciprocal adjustment* we can readily explain this observation in variability of precision in construction. Sometimes we are able to just "count" features, like notice the stepped corbelled ceiling of the Grand Gallery has 7 distinct levels, and there are 28 morticies; that the entrance to the Pyramid on the North face is at the 19 platform (or would be if the missing casing stones were there intact), that there are 203 levels to the pyramid's truncated top, with possibly 216 levels to the apex; that there are 9 granite blocks that form the ceiling of the King's Chamber, that 100 blocks make up the walls of that Chamber, and so on. Gross counting doesn't require any high precision in construction. But, where the architect deliberately used a higher level of precision, we find, when we measure, there are also numeric counts hidden there, encoded in the metrics, requiring that high tolerance. Metrics like the counts, 28 and 40, found hidden in the King's Chamber and the Pyramid's Socket Square at the base, are again repeated as simple counts of features in other parts of the momument. Not only are there 28 mortices along the ramps of the Grand Gallery, but the ceiling of this Gallery is topped off with exactly 40 blocks of stone, confirming the numbers, 28 and 40, found using the method of *reciprocal adjustment* in the measurements of lengths in parts of the design. Taken alltogether, these observations indicate that one function of the Pyramid is to direct attention to certain specific numbers, lengths, and times.

In setting about to establish certain lengths, then, the builders decide on the unit, u, and the count, n, and the three related *unit fractions* of that count, its square, and its cube, $1/n$, $1/n^2$, $1/n^3$, used to determine the three relevant construction parameters, $u = L/n$, $\Delta L = L/n^2$, $tol = L/n^3$, in those parts of the design intended to encode the metrics, (L, u, n). Any surveyor who comes after and attempts to measure the monument, must then consider whether or not he has the measuring ruler accurate enough to measure things within that expected tolerance, $tol = L/n^3$. If the surveyor's ruler doesn't have sufficient precision, he won't even know there is a count, n, there to be found in the length, L !

For example, because the King's Chamber is found to be about $412'' \times 206''$ and the Royal Cubit rod is known to be about $20.6''$ in length, from actual examples of cubit rods archeologists have found, the natural assumption is that the architect of the pyramid intended the King's Chamber to be 20 cubits long by 10 cubits wide. But, as we have seen, by applying the *principle of reciprocal*

adjustment to the actual measurements, the Cubit Rod isn't actually the unit metric used in the construction of the room! It just happens to evenly divide certain multiples of the actual units. The East Wall is measured by a unit rule consisting of 10 digits (a pair of hands), of which there are 28 of these units, or 280 digits, which is 10 cubits because the cubit rod has 28 digits per length. The North Wall is measured by a unit rule consisting of 14 digits (a long span, or half a cubit rod), and there being 40 of these long spans makes 560 digits, which is 20 cubits. The room then uses the long span and pair of hands as the actual primary units, and the fact that the cubit rod fits the dimensions perfectly is just incidental to the design plan.

As discussed, Petrie measured the King's Chamber to, $1''/100 = .01''$, precision, which is not quite adequate to guarantee that both the numbers currently indicated, 28 and 40, are actually the intended divisors of the width and length of the room; the surveyor should measure at least to precision double that, $1''/200 = .005''$, to settle any doubts.[†] Petrie then measures the Socket Square lengths at the base of the Pyramid to precision, $1''/10 = 0.1''$, with error, $\pm.3''$, while here again, the surveyor would do better, if possible, measuring to precision of next higher order, $1''/100 = .01''$.

The problem today with the Socket Square is not so much the availability of accurate measuring instruments, but the ability to determine the exact locations of the points that mark the corners of the square, since the previously well defined features of these sockets have deterioriated substantially since their exposure to the elements from Petrie's day.

However, once the reasoning for choosing these slightly higher tolerance parameters in measuring ruler is understood, the future surveyors, armed with the present theory of the *reciprocal adjustment*, can decide on what characteristics their measuring instruments must have to experimentally confirm or refute the results we have presented.

There's always the risk that one mind sees what it wants to see, because it has been adapted to seeking out patterns, developing biases and preferences in looking, while another mind comes along and sees nothing there at all !

So, when the accuracy of measurements are just at the borderline of a preferred theory, the scientific researcher needs to recognise and acknowledge his own possible bias in interpreting results to fit the theory. The one thing that we can say gives weight to the aforementioned theory, is the apparent *repetition* of the same counts being found encoded in different parts of the momument. So, although each individual metric might be borderline, and questionable, when considered individually, the collection of all the corroborating results acts to increase the probability that the numbers already uncovered would all be verified with any further survey undertaken with higher precision.

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[†]Ideally, the ruler precision used for this room should be $.001''$; but, as previously discussed in "Appendix A - Precision Analysis," p.82, of previous paper [Ref-(1)], a simple change, explained by rounding, $412.53'' - 412.526'' = .004'' \sim .005''$, could also satisfy the thesis.

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